Appendix

A. Analytical presentation of the model

This appendix covers some[19] of the main features of the model. Matrices (uppercase) and vectors (lowercase) are in bold. Parameters are denoted with Greek letters, variables with Latin letters. Unless otherwise stated, all variables and parameters are positive. J is the set of crops indexed by j, and I the set of livestock activities, indexed by i. The constraints defined by $\mathbf{A}_k$, $\mathbf{z}_k$ and $\mathbf{x}_k$ are detailed through decomposing them into sub-matrices and vectors associated with the main technical modules included in the model. Activities (or variables) are presented in columns, and constraints in rows. The dimensions of sub-vectors are given in brackets. Zero values are generally omitted. $\delta_{j,j'}$ is the Kronecker symbol, equal to 1 if $j = j'$ and 0 otherwise. The vector of command variables $\mathbf{x}_k$ is broken down into eight sub-vectors: $\mathbf{y}_k$ (output quantities), $\mathbf{s}_k$ (areas), $\mathbf{a}_k$ (animal numbers), $\mathbf{b}_k$ (purchased quantities of feedstuff), $\mathbf{c}_k$ (on-farm consumption), $\mathbf{v}_k^+$ and $\mathbf{v}_k^-$ (bought and sold live animals live animals, respectively), $\mathbf{d}_k$ (dairy and other animal products). The k-index is omitted and implicit in the rest of the appendix.

A.1. Crop area constraints

Each crop is limited by its maximum area share ($\alpha_j \in [0, 1]$) in total arable land. In addition, the model captures the links between total cereal area and total oilseed area on the one hand, and fodder and meadows on the other hand, through area shares parameters α_{CER}, α_{OIL}, α_{OCE}, and α_{FOD}. Total land endowment is denoted by T.

[19] Given the size of $\mathbf{A}_k$ (approximately 500×1,000), this appendix is not intended to be comprehensive. For further information on the model, the interested reader is referred to `http://www.grignon.inra.fr/economie-publique/MIRAjE/model/detail.htm` and De Cara and Jayet (2000).

Table III. Crop area constraints

	s						**z**
	Cereals	Oilseeds	Other crops	Fodders	Meadows		
	(ha)	(ha)	(ha)	(ha)	(ha)		(ha)
[dim]	$[J_1]$	$[J_2]$	$[J_3]$	$[J_4]$	$[J_5]$		[1]
$[J_1]$	$\delta_{j,j'}/\alpha_j - 1$	-1	-1	-1		$\leq$	0
$[J_2]$	-1	$\delta_{j,j'}/\alpha_j - 1$	-1	-1		$\leq$	0
$[J_3]$	-1	-1	$\delta_{j,j'}/\alpha_j - 1$	-1		$\leq$	0
$[J_4]$	-1	-1	-1	$\delta_{j,j'}/\alpha_j - 1$		$\leq$	0
[1]	$1/\alpha_{CER}$ -1	-1	-1	-1		$\leq$	0
[1]	-1	$1/\alpha_{OIL}$ -1	-1	-1		$\leq$	0
[1]	-1	$1/\alpha_{OCE}$				$\leq$	0
[1]				$1/\alpha_{FOD}$	-1	$\leq$	0
[1]	1	1	1	1	1	$\leq$	T

A.2. Crop output allocation

Crops are divided into three groups: those that can be either sold or on-farm consumed (cereals), those that can only be on-farm-consumed (forage, fodder, pastures, and grassland), and those that are only bound to be sold. The following sub-matrix describes the allocation of total crop production between marketed output and on-farm consumption. Crop yields are denoted by ξ_j.

Table IV. Crop output allocation constraints

	s			**c**	**y**			**z**
	Cereals	Forage	Other crops		Cereals	Other crops		
	(ha)	(ha)	(ha)	(t)	(t)	(t)		(t)
[dim]	$[J_1]$	$[J_4 + J_5]$	$[J_2 + J_3]$	$[J_1 + J_4 + J_5]$	$[J_1]$	$[J_2 + J_3]$		[1]
$[J_1]$	$-\delta_{j,j'}\xi_j$			$\delta_{j,j'}$	$\delta_{j,j'}$		$\leq$	0
$[J_4 + J_5]$		$-\delta_{j,j'}\xi_j$		$\delta_{j,j'}$			$\leq$	0
$[J_2 + J_3]$			$-\delta_{j,j'}\xi_j$			$\delta_{j,j'}$	$\leq$	0

A.3. Animal numbers, feeding and demography

Energy and protein contents of purchased feed (four types of concentrates $q = 1, 2, 3, 4$, and one roughage, $q = 5$) are denoted by $\psi_{i,q,e}$ ($e = 1$ for energy and $e = 2$ for protein). The corresponding parameters for on-farm consumption of cereals are denoted by $\beta_{i,j,e}$, and $\omega_{i,j,e}$ for fodders, forage crops, grassland, and meadows. Energy and protein requirements for animal i are denoted by $\mu_{i,e}$. Maximum quantities of ingested matter are also defined for each

animal i ($\mu_{i,3}$, active for cattle, sheep and goats), as well as the matter contents of the various feedstuffs ($\beta_{i,j,3}$, $\psi_{i,q,3}$, and $\omega_{i,j,3}$).

Table V. Animal feeding constraints

	a	**c**	**b**	**s**		**z**
	Animals	Cereals	Purchased feed	Fodders and meadows		
	(hds)	(t)	(t)	(ha)		
[dim.]	$[I]$	$[J_1]$	[5]	$[J_4 + J_5]$		[1]
$[I \times e = \{1,2\}]$	$\delta_{i,i'}\mu_{i,e}$	$-\beta_{i,j,e}$	$-\psi_{i,q,e}$	$-\omega_{i,j,e}\ \xi_j$	$\leq$	0
$[I]$	$-\delta_{i,i'}\mu_{i,3}$	$\beta_{i,j,3}$	$\psi_{i,q,3}$	$\omega_{i,j,3}\ \xi_j$	$\leq$	0

The cattle demographic module describes the relationships between different age and sex categories (M for males, F for females). The underlying assumption is that demographic equilibrium is achieved. For the sake of compactness, the presentation is limited to three age categories (young, middle, old). Likewise, the presentation does not distinguish between dairy and non-dairy livestock, although the model does. Birth rates are denoted by ρ_i, survival rates by ϵ_i, and adult life span in years by η_i. Milk (and other animal products) yields are denoted by ξ_i. The last two blocks in the following sub-matrix correspond to the possibility for farmers to adjustment their livestock-related capital (stable places). An exponential adjustment at rate ϕ around initial endowment ($\mathbf{a}_0$) is assumed. All rates are annual rates. Animal index i is omitted and implicit in the following sub-matrix.

A.4. Set-aside constraints

The presentation is restricted to the modelling of the different types of set aside, as it provides a good example of how threshold values and integer variables are used in the model. A double system of compensated land set-aside (fixed or rotational) is taken into account. To this regard, important CAP parameters include: reference yields for crop j (ξ_j^r), per-hectare payment (p_j^r), threshold cereal output below which farmers are considered as 'small' crop producers (ζ^s), and the set-aside rate to receive compensated payments (θ^f and θ^r for fixed and rotational set aside, respectively). Distinction is thus made between 'small' crop areas (s_{aj}), large crop areas associated with rotational set aside (s_{bj}), and large crop areas associated with fixed set aside (s_{cj}). Set aside activities are indexed by j ($j = 1, 2$ for rotational and fixed set aside, respectively). κ is a sufficiently large number required by the solving process. p_{sj} refers to per-hectare variable cost. The values $j > 2$ denotes here the rest of the crops. Two binary variables are included (IN and IP) to reflect the either/or nature of the producer choices with respect to set-aside. The constant used in the corresponding constraints is denoted by κ. The first row of the sub-matrix corresponds to the objective function.

Table VI. Animal numbers and demography

Cattle															Other Anim.			
Young (M)			Middle (M)			Old (M)	Young (F)			Middle (F)			Old (F)					
a	$\mathbf{v}^+$	$\mathbf{v}^-$	**a**	$\mathbf{v}^+$	$\mathbf{v}^-$	**a**	**a**	$\mathbf{v}^+$	$\mathbf{v}^-$	**a**	$\mathbf{v}^+$	$\mathbf{v}^-$	**a**	**d**	**a**	**d**	**z**	
1													$-\rho/2$				$= 0$	
-1	-1	1	$1/\epsilon$														$= 0$	
			-1	-1	1	$1/\epsilon$											$= 0$	
							1						$-\rho/2$				$= 0$	
							-1	-1	1	$1/\epsilon$							$= 0$	
										-1	-1	1	$1/(\epsilon\eta)$				$= 0$	
													$-\xi$	1			≤ 0	
															$-\xi$	1	≤ 0	
1	1						1	1										
			1	1														
					1												$\leq (1+\phi)$	$\mathbf{a}_0$
													1					
															1			
1	1						1	1										
			1	1														
					1												$\geq (1-\phi)$	$\mathbf{a}_0$
													1					
															1			

Table VII. Set-aside constraints

'Small' producers		Rotational set aside		Fixed set aside		IN	IP		**z**
s_a		s_b		s_d					
$j=1$	$j>2$	$j=1$	$j>2$	$j=2$	$j>2$				
$-p_{sj}+p_j^r$	$-p_{sj}+p_j^r$	$-p_{sj}+p_j^r$	$-p_{sj}+p_j^r$	$-p_{sj}+p_j^r$	$-p_{sj}+p_j^r$				(=OBJ)
ξ_j^r	ξ_j^r							$\leq$	ζ^s
		$-1+\theta^r$	θ^r					$\leq$	0
				$-1+\theta^f$	θ^f			$\leq$	0
1	1					$-\kappa$		$\leq$	0
		1	1	1	1		$-\kappa$	$\leq$	0
						1	1	$=$	1